P425/1

PURE

MATHEMATICS

Paper 1

Jul/Aug 2019

3 Hours

MUKONO EXAMINATION COUNCIL

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 Hours

INSTRUCTIONS TO CANDIDATES

*Answer **all** the eight questions in section **A** and any **five** from section **B***

*Any addition question(s) answered will **not** be marked*

*All necessary working **must** be clearly shown*

Begin each answer on a fresh sheet of paper

Silent, non – programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 marks)

*(Answer **all** questions in this section)*

1. Solve for x, in the equation $9^{x-1} - 3^{x+2} + 162 = 0.$ ***(5 marks)***

2. The lines $4x - 3y = 5$ and $y = 3$ are tangents to two circles whose centres lie on the line $x = 7$. Find the distance between the centres of the circles. ***(5 marks)***

3. Solve $sec^2(2\theta) - 3tan2\theta + 1 = 0\,,\ \ for\ \ 0^0 \leq \theta \leq 180^0$ ***(5 marks)***

4. The ages of a mother and her three children are in a geometrical progression, the sum of their ages is 195 years and the sum of the ages of the two young children is 60 years. Find the age of the mother. ***(5 marks)***

5. Evaluate $\int_3^5 \frac{2(x+1)}{2x^2-3x+1}\, dx.$ ***(5 marks)***

6. The equation of the normal to the curve $xy^2 + 3y^2 - x^3 + 5y - 2 = 0$ at the point $(a, -2)$ is $15x - 8y - 46 = 0$. Find the value of a. ***(5 marks)***

7. Find $\frac{dy}{dx}$ if $y = xsin^2x$. when $x = \frac{\pi}{4}$ ***(5 marks)***

8. Find the Cartesian equation of a plane containing point (1, 3, - 4) and the line $\frac{x-1}{2} = \frac{y+2}{3} = z.$ ***(5 marks)***

SECTION B (60 marks)

(Answer any **five** questions from this section. All questions carry equal marks)

9. (a.) Given that $2A + B = 135$ show that $tanB = \frac{tan^2A - 2tanA - 1}{1 - 2tanA - ta\ \ ^2A}$. ***(4 marks)***

   (b.) If $\alpha$is an acute angle and $tan\alpha = \frac{4}{3}$, show that $4\sin(\theta + \alpha) + 3\cos(\theta + \alpha) = 5cos\theta$. Hence solve for θ the equation $4\sin(\theta + \alpha) + 3\cos(\theta + \alpha) = \frac{\sqrt{300}}{4}$ for $-180^0 \leq \theta \leq 180^0$. ***(8 marks)***

10. (a.) Show that $y = x - 3$ is a tangent to the curve $y = x^2 - 5x + 6$. ***(3 marks)***

    (b.) A chord to the parabola $4x - 3y^2 = 0$ is parallel to the line $2x - y = 4$ and passes through point (1, 1). Find;

    (i.) the equation of the chord.

    (ii.) the coordinates of the points of intersection of the chord with the parabola.

    (iii.) the acute angle between the chord and the directrix of the parabola.

    (9 marks)

11. (a.) Expand $(4-3x)^{\frac{1}{2}}$ in ascending powers of x up to the term in x^3. Taking $x=\frac{1}{25}$ find $\sqrt{97}$. ***(8 marks)***

(b.) Find the term independent of x in the binomial expansion of $\left(2x-\frac{1}{x^2}\right)^9$.

(4 marks)

12. (a.) Solve for x and y values in the equation; $\frac{x}{2+3i}+\frac{y}{3-i}=\frac{6-13i}{9+7i}$. ***(6 marks)***

(b.) Given that $-4+i$ is a root of the equation $z^4+6z^3+6z^2+6z+65=0$, find the other roots of the equation and represent the roots in polar form. ***(6 marks)***

13. (a.) Find the volume of a solid generated by rotating about the y-axis, the area enclosed by the curve $y^2+4x=9$, the y-axis and $y=-2$. ***(5 marks)***

(b.) Find $\int x\ln(2x)dx$. ***(3 marks)***

(c.) Evaluate $\int_0^1 \frac{2x-1}{(x-3)^2}\,dx$. ***(4 marks)***

14. The points A, B, C and D are given by the coordinates (5, 2, -3), (-1, 0, -1), (9, 5, -8) and (5, 7, -14) respectively. If lines AB and CD intersect at point E. Find;

(i.) Equations of lines AB and CD.

(ii.) Coordinates of point E

(iii.) The acute angle between lines AB and CD. ***(12 marks)***

15. A curve is given by the parametric equations; $x=3t$and $=\frac{2t^2}{1-t}$.

(a.) Find the Cartesian equation of the curve.

(b.) Sketch the curve, showing clearly the asymptotes and turning points.

(12 marks)

16. (a.) Solve the differential equation $\frac{dy}{dx}=4x-7$, given that $y(2)=3$. ***(3 marks)***

(b.) The rate at which a candidate was losing support during an election campaign was directly proportional to the number of supporters he had at that time. Initially he had V_o supporters and t weeks later, he had V supporters.

(i.) Form a differential equation connecting V and t.

(ii.) Given that the supporters reduced to two thirds of the initial number in 6 weeks, solve the equation in (i.) above.

(iii.) Find how long it will take for the candidate to remain with 20% of the initial supporters. ***(9 marks)***

End